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The Mathematical modeling of Pranic Body

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Abstract

In Yogic anatomy, each of us has nine bodies other than the physical one we are well-acquainted with. One of those bodies is called Pranic body. Here by Pranic body is to be thought as the breath which we all intake. In this paper we developed a mathematical model for different breathing pattern for unsteady-state equations. All these equations are time dependent under constant metabolic rate. A compartment model of breath function from lungs to tissues is discussed and we have also discussed the analytical solution of breath expressions. The Variation of the concentration of oxygen in heart, lungs, and in body for normal breath is shown in fig (1.1), fig (1.2), and in fig (1.3).

Key words: Anatomy, steady-state, metabolic rate

Introduction

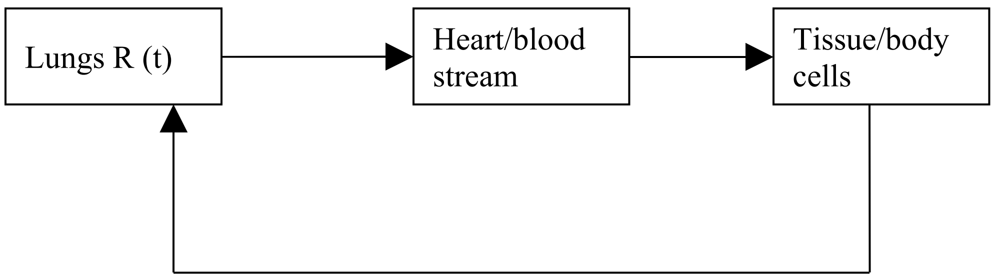
Pranayama is an important, yet little known part of Yoga. Its techniques have been practiced for centuries by ardent students of yoga in remote ashrams, and have been presented for us through many generations both in practice and in hand written books.

This art and science of yogic breathing was almost completely unknown to the conman man like many other ancient Indian arts.

The modeling of Pranic body discussed by Brain Bergen (2006) by using steady –state condition to the governing equation [3] and also discusses the behavior of lung, heart, body cells and breathes function for one minute breath.

In this paper we have developed a compartment model of breath function from lungs to tissues and we discussed the analytical solution of breath expressions. The main assumption for the first model we constructed that the oxygen enters and exits by the lungs with breath function R (t), with unit volume/time and then transferred to the heart/bloodstream at a rate that depends on the amount available in the lungs and the amount already in the blood. Since it is possible that there is still oxygen left in the lungs (indeed there is a minimal volume of about 1.2L, and the air that gets caught in the bronchial tree at the end of the inhale will be the first to be exhaled [5].) The change in the concentration of oxygen in the lungs due to exhaling is also considered. We assume that the flow from lungs to the blood depends on the concentration already in the blood since oxygen binds to the hemoglobin in the blood. In reality, this is a non- linear relationship [5], but for simplicity, it is assumed that it is directly proportionate to the difference in concentrations of the two chambers. So we assumed that oxygen flows from the blood to the cells of the body and Vice-Versa. Finally, the oxygen is consumed by the body at a rate proportionate to the amount available and the metabolic rate M. [3]

Mathematical Model & Equations



$$\frac{dL}{dt} = \frac{-\sqrt{R(t)^2}}{V_{\max}}.L - \eta\,(L - H) + RQ; \tag{1.1}$$

$$\frac{dH}{dt} = \eta\,(L - H) - f(H - B) \tag{1.2}$$

$$\frac{dB}{dt} = f(H - B) - MB \tag{1.3}$$

where

$$R = \frac{A}{2}\sin\left(\frac{2\pi t}{p}\right) \tag{1.4}$$

$$Q = Q_{\max}\{2\sin^2\frac{\pi .t}{\tau} - 1\} \tag{1.5}$$

Variables and Parameters

L ≡ concentration of oxygen in lungs.

H ≡ concentration of oxygen in heart/blood.

B ≡ concentration of oxygen in body/cells.

M ≡ metabolic rate – how much oxygen is being used by the body.

R ≡ breathe function. Inhale R>0

Exhale R<0

η – Maximum rate of transfer of molecules from the lungs to the bloodstream.

f – Maximum rate of transfer of molecules from the bloodstream to the body cells.

V_{max} – Maximum lung volume.

p – Period of breath.

τ – It is also the breathing period.

$Q_{max} < 24$ L/min, amount of oxygen.

All the concentration to make sense, L$\geq$ 0, H$\geq$ 0, B$\geq$ 0.

Analytical Solutions of the Equations

Solving the above simultaneous differential equation(1.1-1.3) and we get the values of L, B, and H such as

$$B = \frac{\frac{A}{2}\sin\left(\frac{2\pi}{p}.t\right).Q_{max}\left(2.\sin^2\frac{\pi .t}{\tau}\right)\eta .f}{(f+M)\left[\left(\frac{\eta + f}{V}\right).\frac{A}{2}.\sin\left(\frac{2\pi .t}{p}\right)+\eta .f\right]+f^2} \tag{1.6}$$

$$H = \frac{[\eta .A.Q_{\max}(t_1 + t_2) + RQ\eta f(f + M)].t_3 - \frac{\eta\ \pi\ .A.R.Q(f + M)}{p}\cos\frac{2\pi\ .t}{p}}{t_3^2} \tag{1.7}$$

Where

$$t_1 = \frac{1}{p}.\cos\frac{2\pi\ .t}{p}.\left(2\sin^2\frac{\pi\ .t}{\tau} - 1\right)$$

$$t_2 = \frac{1}{\tau}\sin\frac{2\pi\ .t}{p}.\sin\frac{2\pi\ .t}{\tau}$$

$$t_3 = (f + M)\left(\frac{\eta + f}{V}.R + \eta\ .f\right) + f^2$$

And the concentration of oxygen in lungs

$$L = \frac{t_3^2[\eta\ .A\pi\ .Q_{\max}(t_4 + t_5 + t_6 + t_7).t_3 + t_8(\eta\ .A\pi\ .Q_{\max}.(t_1 + t_2) + t_9] - \alpha\,[t_1 + \tau\ .t_2] - \beta\ .t_3\delta}{t_3^4} + (\eta + f).H + f.B$$

Where the constant symbols used as: (1.8)

$$t_4 = \frac{1}{p}\cos\frac{2\pi\ .t}{p}\left(\frac{4\pi}{\tau}\sin\frac{\pi\ .t}{\tau}.\cos\frac{\pi\ .t}{\tau}\right)$$

$$t_5 = \frac{2\pi}{p^2}\sin\frac{2\pi\ .t}{p}\left(2\sin^2\frac{\pi\ .t}{\tau} - 1\right)$$

$$t_6 = \frac{2\pi}{\tau}\left(\frac{1}{p}\cos\frac{2\pi\ .t}{p}\sin\frac{2\pi\ .t}{\tau} + \frac{1}{\tau}\sin\frac{2\pi\ .t}{p}\cos\frac{2\pi\ .t}{\tau}\right)$$

$$t_7 = \eta\ .f(f + M)\left\{\frac{\pi A.Q}{p}\cos\frac{2\pi\ .t}{p} + \frac{2\pi\ .R.Q_{\max}}{\tau}\sin\frac{2\pi\ .t}{\tau}\right\}$$

$$t_8 = \frac{(f + M)(\eta + f)}{V}.\frac{\pi\ .A}{p}.\cos\frac{2\pi\ .t}{p}$$

$$t_9 = RQ\eta . f(f + M)$$

$$\alpha = \frac{\pi^2 . A^2 \eta}{2}(f + M)Q_{\max}$$

$$\beta = [\eta A\pi . Q_{\max}(t_1 + t_2) + t_9].t_3 - \frac{\eta \pi . A.R.Q(f + M)}{p}.\cos\frac{2\pi .t}{p}$$

$$\delta = \frac{(\eta + f)}{V}\frac{A\pi}{p}(f + M)^2\left[\frac{(\eta + f)}{V}.R + \eta f + f^2\right].\cos\frac{2\pi .t}{p}$$

The constant values of the included parameters such as $\eta, Q_{\max}, V, f, M$ are 0.1/s, 24L/min, 3600ml, 0.7/s, and 0.01/s.

Analysis

If the function R does not depend on time, then the system becomes autonomous, as in the case, where one is forever inhaling or exhaling.

When R=0, Q=0 and R>0. So, as expected, the concentration of oxygen in the body goes to 0. If one is always exhaling. More interesting is the case for R>0. The breath function R is linearly depends upon the concentration of oxygen. The basic results obtained from the equations are:

R	**B predicated**	**B actual**
1000	0.02796936193810	0.02796936193801
500	0.02712058072322	0.02712058069406
250	0.02556872188074	0.02556870891066

Results

In order to generate the following results, we used the following parameters,
V=3600ml, η = 0.1/sec, f =0.7/sec, M=0.01/sec
For the normal breath function, R was calculated from the tidal volume (about 500ml) and the maximum lung volume was used and the breath rate is defined as 20sec inhale, 20sec suspension, 20 sec exhale.

We observed the functions L, H, and B (Concentration of oxygen in lungs, heart and body for the normal breath in fig1.1. Compare it to the one minute breath function, in fig 1.2. Though it appears that the one minute breath hasn't a chance to reach equilibrium. After only one minute, it seems similar to the normal breath in value, except there is some smoothing in the amount of oxygen in the lungs. The breath of fire pattern, however, trumps the other two. The breath of fire reaches a non-oscillatory equilibrium.
For the Pranic body**,** from the fig (1.1) it is clear that the concentration of oxygen in lungs (when we inhale), first it will increase continuously, after a time period it will become constant i.e. the state of saturation. The condition is fulfilled also for the exhalation i.e. when we release oxygen from our body first it will decrease then after some time period this is continuous, this state is also called state of saturation. From fig (1.1), in the body cells the oxygen falls periodically and becomes constant for one minute.

In fig (1.2), initially the concentration of oxygen in lungs is very low, as soon as time increases concentration increases up to a maximum value and rapidly falls down to the starting point. After that concentration becomes approximately constant during one minute time interval in deep breathing position. Similarly the concentration of oxygen in body increases initially and becomes approximately constant. The concentration of heart falls down initially and becomes approximately constant during one minute time interval in deep breathing position.

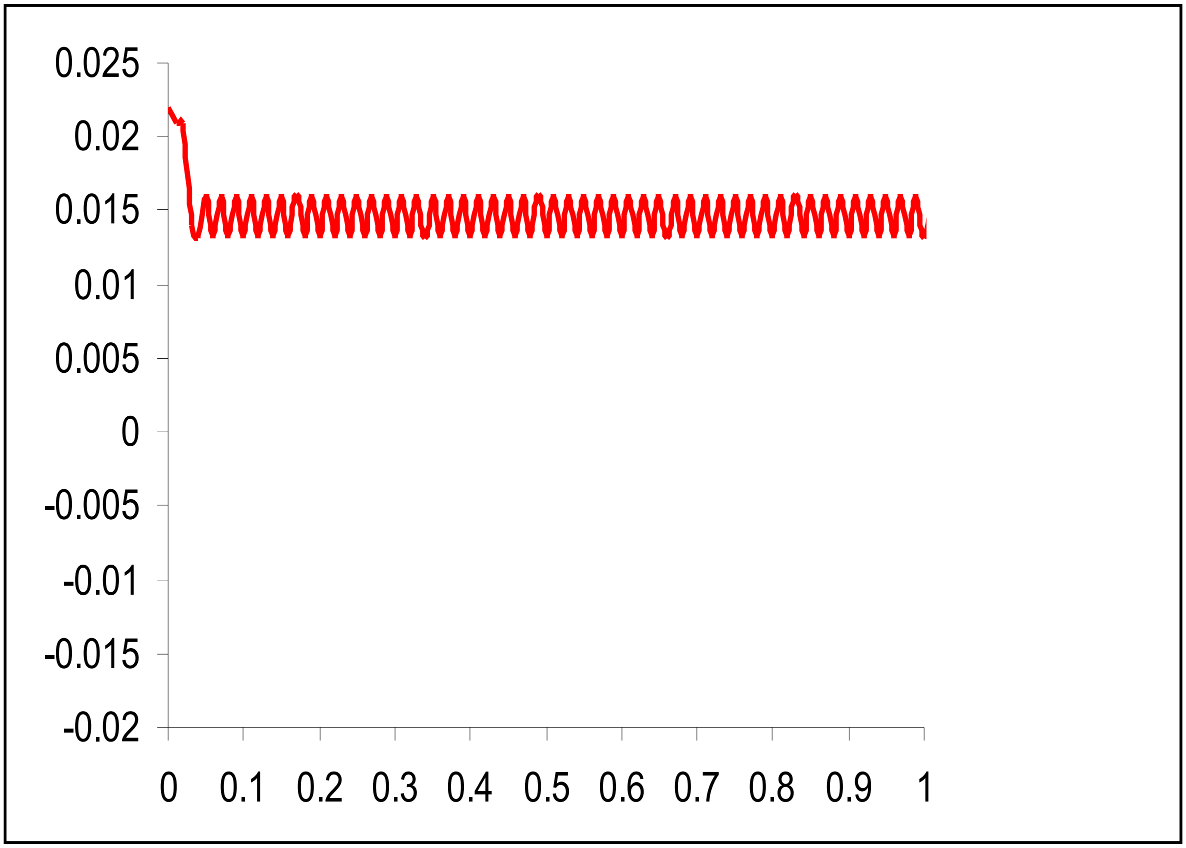


Fig (1.1): Normal breathing simulation for one minute.
(X-axis shows time, Y-axis shows L, B, H functions)

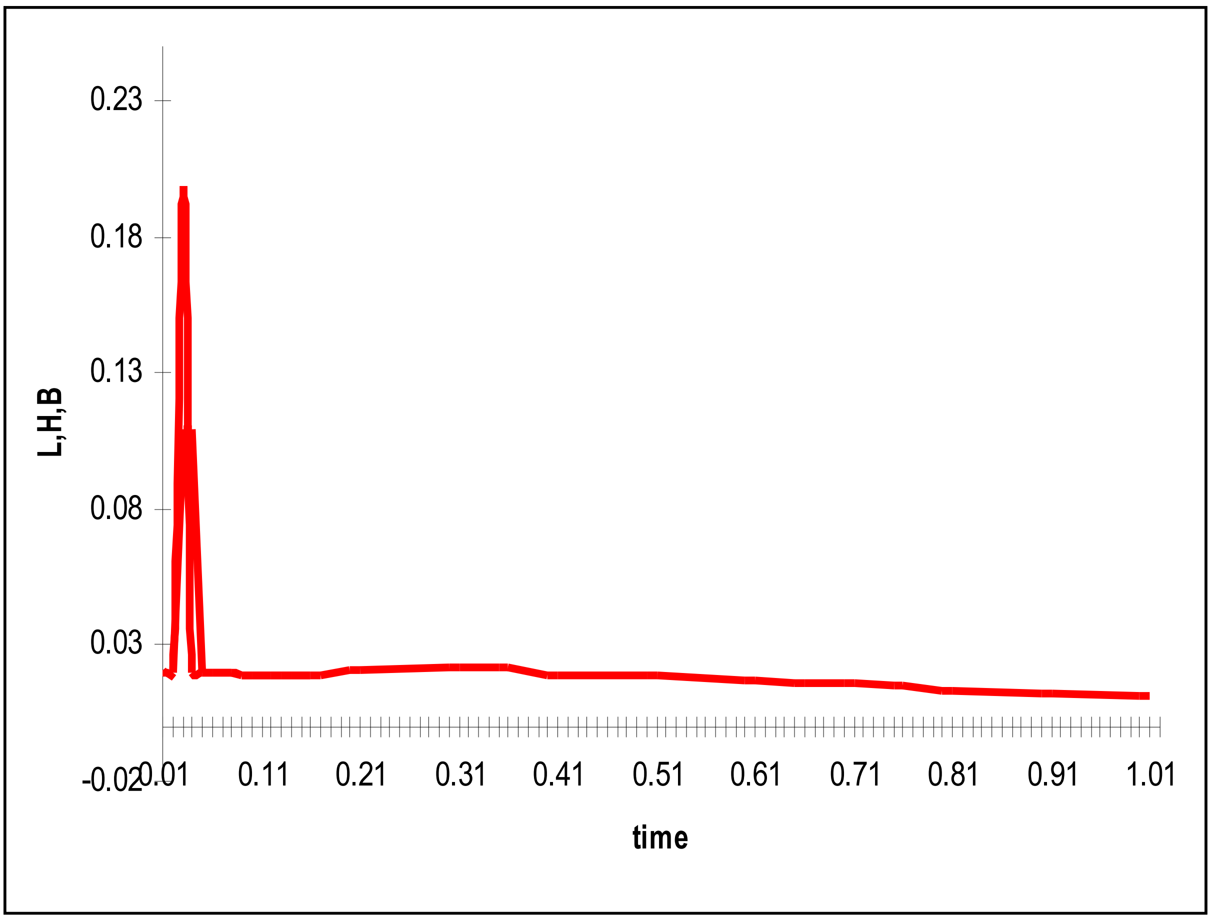


Fig (1.2): one minute breathe simulation

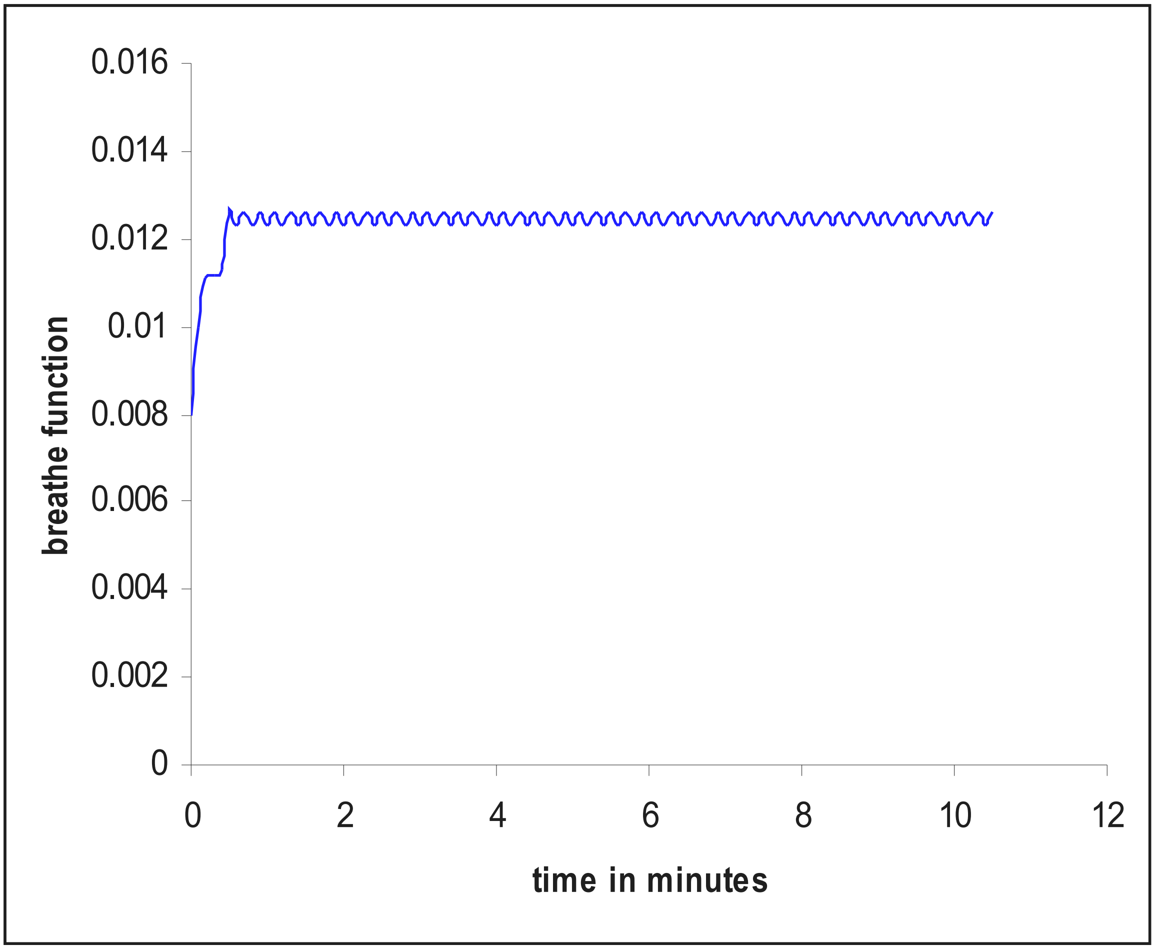


Fig (1.3): Comparison of three breathing patterns (normal, one minute, Breath of fire) over 11 minutes.

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